Let *X* be a variable random represent number of defect item in each sample.

Let is the observation of number of defect item in sample i-th size p ( represent the i-th sample). Let *n* is number of sample size *p*.

Suppose that are auto-correlated and follow the identical normal distribution with known in control mean and standard deviation . The process can be modelled as follow by AR(1) model:

where is parameter of AR(1).

Suppose that the mean is shifting from to where is magnitude of mean shift in term of .

Note that the number of defects in each sample size n only concern that whether the sample is conforming (whether the number of defects which *Lu* is the upper warning limit and *Lu* can be expressed:

which *k* is warning limit coefficient

Let is probability that sample i-th is non-conforming sample, i.e., sample has number of defect item larger than in state j. we use in this study indicates the process in control and out of control respectively.

Let is CDF of standard normal distribution, and

is PDF of standard normal distribution.

Define is probability that number of nonconforming item in sample *i*-th larger than *Lu* in state-*j*.

When i = 1

When i = 2

In general:

Thus, we can write that:

1. When i =1

………………………………… (1)

1. When

………………….. (2)

which:

We found that the probability depends on as result of the autocorrelation property shown in equation (2).

Since the classification of conforming and non-conforming samples is a Bernoulli trial, we define as a sequence of Bernoulli random variable for a fixed such that:

Let denote the statistic sample of a CCCG chart, i.e. the cumulative number of samples size-p inspected until the first non-conforming sample is encountered. Then, is generally considered to be a geometric random variable with parameter .

Let be the probability that the total number of conforming samples smaller than *LCL* in state-*j*, i.e., the probability that control chart produces out-of-control signal when the process is in state-*j*.

with

The probability of type I error is and the probability of type II error is

Therefore,

As it is difficult to derive the closed-form solution for ARL, we present a step-by-step description of procedure for calculating ARL in the following table:

|  |  |
| --- | --- |
| Set |  |
| Compute  Set and  [cycle *i*] For *i* = 2 to *n* step 1  Compute  Compute and  Next [cycle *i*]  Compute and  Compute and  Stop | Through Eq. 1  Trough Eq. 2 |

Genetic Algorithm: